Haverford Problem Solving Group
September 14, 2023
Geometry

Welcome to the Problem Solving Group, a.k.a. PSG! All materials will be posted to www.guilhermezeus.com/psg. Scan the QR code to join our mailing list.

G1. One of the cross section in a rectangular box is a regular hexagon. Prove that the box is a cube. [Eng98, p. 318]

G2. Can two triangles have two equal dised and three equal angles, and still be noncongruent? If yes, then give conditions. [Eng98, p. 318]

G3. Let P be a point inside a continuous closed curve in the plane which does not intersect itself. Show that there are two points on the curve whose midpoint is P. [Sch99] [Loh22]

G4. What is the maximum area of a quadrilateral with sides 1, 4, 7, 8? [Eng98, p. 319]

G5. Any four of five circles have a common point. Prove that all five circles have a common point. [Eng98, p. 321]

G6. Given any bounded plane region, prove that there are three concurrent lines that cut it into six pieces of equal area. [Loh22]

G7. Using only an unmarked ruler, solve the following items. **(a)** Given two paralel segments, construct their midpoints. **(b)** Given a segment s, its midpoint, and a point $P \notin s$, construct a line r parallel to s through P. **(c)** Given a paralellogram, draw a parallel through its center to a side. [Eng98, pp. 319–320]

For more problems, turn the page.

G8. Let convex quadrilateral $ABCD$ be given in a plane, and let X be a point not on the plane. Show that there are points A', B', C', and D' on the lines XA, XB, XC, and XD, respectively, with the property that $A'B'C'D'$ is a parallelogram. [Loh22]

G9. (a) Given a finite collection of closed squares of total area 3, prove that they can be arranged to cover the unit square. **(b)** Given a finite collection of closed squares of total area $\frac{1}{2}$, prove that they can be arranged to fit in the unit square (with no overlaps). [Loh22]

G10. Let OA and OB be two rays in the plane, and let P be a point between them. Which point X on the ray OA has the property that if XP is extended to meet the ray OB at Y, then $XP \cdot PY$ is minimized? [Loh22]

G11. Suppose that the sun is exactly overhead. How should I hold a rectangular box over a horizontal table so that its shadow has maximum area? [Eng98, p. 321]

G12. Given a region whose boundary is a simple polygon of area a and perimeter p, prove that it contains a disc with radius larger than a/p. [Loh22]

G13. Given a right triangle and a finite set of points inside it, prove that these points can be connected by a path of line segments, such that the sum of squares of segment lengths in this path is at most the square of the hypotenuse. [Loh22]

G14. Let an ellipse have center O and foci A and B. For a point P on the ellipse, let d be the distance from O to the line of tangency to the ellipse at P. Show that $PA \cdot PB \cdot d^2$ is independent of the position of P. [Loh22]

References

[Eng98] Arthur Engel. *Problem-Solving Strategies*. Problem Books in Mathematics. New York: Springer, 1998.

[Loh22] Po-Shen Loh. *Geometry*. Carnegie Mellon University Putnam Seminar. 2022. URL: https://www.math.cmu.edu/~ploh/docs/math/2022-295/13-geometry.pdf.

[Sch99] John Scholes. *Putnam 1977/B4 Solution*. Nov. 30, 1999. URL: https://prase.cz/kalva/putnam/psoln/psol7710.html.